SCREENING AND ABSORPTION OF GRAVITATION IN PRE-RELATIVISTIC AND RELATIVISTIC THEORIES

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Abstract. After commenting on the early search for a mechanism explaining the Newtonian action-at-a-distance gravitational law we review non-Newtonian effects occurring in certain ansatzes for shielding, screening and absorption effects in pre-relativistic theories of gravity. Mainly under the aspect of absorption and suppression (or amplification), we then consider some implications of these ansatzes for relativistic theories of gravity and discuss successes and problems in establishing a general framework for a comparison of alternative relativistic theories of gravity. We examine relativistic representatives of theories with absorption and suppression (or amplification) effects, such as fourth-order theories, tetrad theories and the Einstein-Cartan-Kibble-Sciama theory.

1. Introduction

All deviations from the gravitational theories of Newton and Einstein touch fundamental problems of present-day physics and should be examined experimentally. In particular, such examination provides further tests of Einstein's general theory of relativity (GRT), which contains Newton's theory as an approximate case. Therefore, it makes sense to systematically analyze all effects that differ from the well-known Newtonian and post-Newtonian ones occurring in GRT (let us call them non-Einsteinian effects). If these effects can be excluded experimentally, then this would provide further support for GRT; otherwise one would have to change basic postulates of

present-day physics. Of all possible non-Einsteinian effects, we shall focus in this paper on the effects of shielding, absorption and suppression of gravitation.

The experience made with GRT shows that it is not probable to find any non-Einsteinian solar-system effects. Nevertheless, it is useful to search for them on this scale, too, for it can establish further null experiments in support of GRT. However, a strong modification of GRT and corresponding effects can be expected on the microscopic and, possibly, cosmological scale. As to the microscopic scale, this conjecture is insofar suggesting itself as there is the problem of quantum gravitation unsolved by GRT. The cosmological part of this conjecture concerns the standpoint with respect to the Mach principle. Of course, there is no forcing physical argument for a realization of this principle, and GRT did not leave unanswered the question as to it. But one can be unsatisfied with this answer and look for a generalization of GRT satisfying this principle more rigorously than GRT. If this is the case, one has a further argument for a modification of GRT predicting also non-Einsteinian effects. (In [12], it was shown that the tetrad theories discussed in Sec. 3.2 could open up new perspectives for solving both problems simultaneously.)

We begin in Sec. 2 with remarks on the early search for a mechanism explaining the Newtonian action-at-a-distance gravitational law and then discuss non-Newtonian effects occurring in certain ansatzes for shielding, screening and absorption effects in pre-relativistic theories of gravity. In Sec. 3, mainly under the aspect of absorption and suppression (or amplification), we consider successes and problems in establishing a general framework for a comparison of alternative relativistic theories of gravity. We discuss then the following relativistic representatives of theories with such effects, such as: in Sec. 3.1 fourth-order derivative theories of the Weyl-Lanczos type formulated in Riemann space-time, in Sec. 3.2 tetrad theories formulated in teleparallelized Riemann space-time, and in Sec. 3.3 an effective GRT resulting from the Einstein-Cartan-Kibble-Sciama theory based on Riemann-Cartan geometry. Finally, in Sec. 4 we summarize our results and compare them with GRT. The resulting effects are interpreted with respect to their meaning for testing the Einstein Equivalence Principle and the Strong Equivalence Principle.

2. Shielding, absorption and suppression in pre-relativistic theories of gravity

To analyze non-Einsteinian effects of gravitation it is useful to remember pre-relativistic ansatzes of the nineteenth century, sometimes even going back to pre-gravitational conceptions. One motivation for these ansatzes

was to find a mechanical model that could explain Newton's gravitational inverse-square-law by something (possibly atomic) that might exist between the attracting bodies. In our context, such attempts are interesting to discuss, because they mostly imply deviations from Newton's law. Another reason for considering such rivals of Newton's law was that there were several anomalous geodesic, geophysical, and astronomical effects which could not be explained by Newtonian gravitational theory. Furthermore, after the foundation of GRT, some authors of the early twentieth century believed that there remained anomalies which also could not be explained by GRT. As a result, pre-relativistic assumptions continued to be considered and relativistic theories competing with GRT were established.

One influential early author of this story was G.-L. Le Sage [79, 80]. In the eighteenth century, he proposed a mechanical theory of gravity that was to come under close examination in the nineteenth century (for details on Le Sage's theory, see also P. Prevost [105] and S. Aronson [2]). According to this theory, space is filled with small atomic moving particles which due to their masses and velocities exert a force on all bodies on which they impinge. A single isolated body is struck on all sides equally by these atoms and does not feel any net force. But two bodies placed next to each other lie in the respective shadows they cast upon each other. Each body screens off some of the atoms and thus feels a net force impelling it toward the other body.

Under the influence of the kinetic theory of gases founded in the 1870's Le Sage's theory was revived by Lord Kelvin [118][1], S. T. Preston [103, 104], C. Isenkrahe [64] and P. Drude [33], bringing Le Sage's hypothesis up to the standard of a closed theory. However, this approach to gravitation was rejected by C. Maxwell [90] with arguments grounded in thermodynamics and the kinetic theory of gases. On the basis of these arguments, it was discussed critically by Poincaré and others (for the English and French part of this early history, cf. Aronson [2]).

The search for a mechanistic explanation of gravitation and the idea of a shielding of certain fluxes that intermediate gravitational interaction were closely related to the question of the accuracy of Newton's gravitational law. In fact, this law containing only the masses of the attracting bodies and their mutual distances can only be exactly valid when neither the intervening space nor the matter itself absorb the gravitational force or potential. Therefore, it is not surprising that the possibility of an absorption of gravitation had already been considered by Newton in his debate with E. Halley and N. Fatio de Duillier[2], as is documented in some of the Queries to Newton's "Opticks." The first research program looking for an experi-

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[1]Later he reconsidered it from the view of radioactivity [119].
[2]Le Sage himself stated that his speculations go also back to the work of Fatio.

mental answer to this question was formulated by M.W. Lomonosov [83] in a letter to L. Euler in 1748. The program was however only realized 150 years later by R. von Eötvös and Q. Majorana, without explicit reference to Lomonosov. At about the same time, Euler discussed with Clairaut, a prominent specialist in celestial mechanics, the possibility of detecting deviations from Newton's gravitational law by analyzing the lunar motion. Clairaut believed for a time that he had found a fluctuation of the lunar motion testifying to an absorption of gravitation by matter, in this case, by the earth.

For a long time, the lunar motion has been the strongest criterion for the validity of the Newtonian and, later, the Einsteinian theory of gravity (today one would study the motions of artificial satellites). This is due to the fact that the gravitational influence of the sun on the moon exceeds the influence of the earth by the factor 9/4. This solar action varies in dependence on the distance of the system 'earth-moon' from the sun. Regarding this effect and, additionally, the action of the other planets on the lunar motion, a reaming fluctuation of the motion of the moon could possibly be due to an absorption of solar gravity when the earth stands between the sun and the moon. This early idea of Euler was later revived by von Seeliger, and just as Clairaut had analyzed the lunar motion in order to corroborate it, later Bottlinger [21, 22] did the same in order to find support for the hypothesis of his teacher von Seeliger [114].

The first ansatz for an exact mathematical description of absorption in the sense of Euler and Lomonosov was made by Laplace [76] in the last volume of his *Mécanique Céleste*. He assumed that the absorption $d\vec{F}$ of the flow $\vec{F}$ of the gravitational force is proportional to the flow $\vec{F}$ itself, the density ρ, and the thickness dr of the material penetrated by the gravitational flow, $d\vec{F} = k\rho\vec{F}dr$. Accordingly a mass element dm_1 exerts on another element dm_2 the force

$$|d\vec{F}| = \frac{Gdm_1dm_2}{r^2}e^{(-k\rho r)}, \tag{1}$$

where k is a universal constant of the dimension $(mass)^{-1}(length)^2$ and G is Newtons gravitational constant.

In the early twentieth century, when Newton's gravitational theory was replaced by GRT, the two aforementioned attempts by Bottlinger and Majorana were made to furnish observational and experimental proof of absorption effects in the sense of Euler and von Seeliger. Such effects do not exist in GRT and so evidence for them would have been a blow against the theory.

Using H. von Seeliger's hypothesis of 1909 (von Seeliger [114]), F.E. Bottlinger [21, 22] tried to explain short-period fluctuations of the motion

of the moon (later it became clear that this explanation was not correct), while Majorana attempted to detect such absorption effects by laboratory experiments from 1918 till 1930. Being aware of previous experiments performed to detect an absorption of gravitation by matter, Majorana turned to this problem in 1918. He speculated that gravitation was due to a flow of gravitational energy from all bodies to the surrounding space which is attenuated on passing through matter. The attenuation would depend exponentially on the thickness of the matter and its density. Based on a theoretical estimation of the order of magnitude of this effect he carried out experiments the results of which seemed to confirm the occurrence of gravitational absorption. According to present knowledge, they must have been erroneous (for details of the history of these experiments, see, e.g., Crowley et al. [27], Gillies [55], Martins [1]).

Another conception competing with absorption or shielding of gravitation by matter also goes back to papers by von Seeliger [113]. In these papers, now for cosmological reasons, he considered a modification of Newton's law by an exponential factor. Similar ideas were proposed by C. Neumann [96]. At first sight, it would appear to be the same modification as in the former cases. This is true, however, only insofar as the form of the gravitational potential or force is concerned. For the differential equations to which the respective potentials are solutions, there is a great difference. In the first case, instead of the potential equation of Newtonian theory,

$$\triangle\phi = 4\pi G\rho, \tag{2}$$

one has equations with a so-called potential-like coupling,

$$\triangle\phi = 4\pi G\rho\phi, \tag{3}$$

while in the second case one arrives at an equation with an additional vacuum term,

$$\triangle\phi - k^2\phi = 4\pi G\rho. \tag{4}$$

The latter equation requires the introduction of a new fundamental constant k corresponding to Einstein's cosmological constant Λ. As will be seen in Sec. 3.2, in relativistic theory a combination of vacuum and matter effects can occur, too.

In [122] it is shown that, regarding cosmological consequences, both approaches are equivalent. Indeed, if one replaces in (3) ϕ by $\phi + c^2$ and considers a static cosmos with an average matter density $\bar{\rho} = const$ then the corresponding average potential $\bar{\phi}$ satisfies the equation

$$\triangle\bar{\phi} - k^2\bar{\phi} = 4\pi G\bar{\rho} \tag{5}$$

where

$$k^2 = 4\pi G \bar{\rho} c^{-2} \tag{6}$$

which is identical to the averaged equation (4).

Equation (3) shows that the potential-like coupling of matter modelling the conception of an absorption of the gravitational flow by the material penetrated can also be interpreted as a dependence of the gravitational number on the gravitational potential and thus on space and time. Therefore, to some extent it models Dirac's hypothesis within the framework of a pre-relativistic theory, and a relativistic theory realizing Dirac's idea could have equation (3) as a non-relativistic approximation. Another possible interpretation of (3) is that of a suppression of gravitation (self-absorption) by the dependence of the active gravitational mass on the gravitational potential. Indeed, the product of the matter density and the gravitational potential can be interpreted as the active gravitational mass.

Not so much for historical reasons but for later discussion, it is interesting to confront the potential equations (2) and (4) with the bi-potential equations [18],

$$\triangle \triangle \phi = 4\pi G \rho \tag{7}$$
$$\triangle(\triangle - k^2)\phi = 4\pi G \rho \tag{8}$$

or, for a point-like distribution of matter described by the Dirac delta-function δ,

$$\triangle \triangle \phi = 4\pi a \delta(\vec{r}) \tag{9}$$
$$\triangle(\triangle - k^2)\phi = 4\pi a k^2 \delta(\vec{r}). \tag{10}$$

Eqs. (9) and (10) show that the elementary solutions are given by the Green functions,

$$\phi = \frac{ar}{2} \tag{11}$$
$$\phi = \frac{a}{r} - \frac{a}{r}e^{-kr} \tag{12}$$

indicating that there is a long-range (Newtonian) and a short-range (self-absorption) part of gravitational interaction.

In order to select these physically meaningful solutions from a greater manifold of solutions for the fourth-order equations one had to impose yet an additional condition on the structure of sources: The distribution of matter must be expressed by a monopole density. This becomes obvious

when one considers the general spherical-symmetric vacuum solutions of
(2, 10) which are given by

$$\phi = \frac{ar}{2} + b + \frac{c}{r} + \frac{d}{r^2} \qquad (13)$$

$$\phi = \frac{A}{r} + B + C\frac{e^{-kr}}{r} + D\frac{e^{kr}}{r} \qquad (14)$$

respectively, satisfying the equations

$$\triangle \triangle \phi = -4\pi a \delta(\vec{r}) - 4\pi c \triangle \delta(\vec{r}) \qquad (15)$$

$$\triangle(\triangle - k^2)\phi = -4\pi(A + C) \triangle \delta(\vec{r}) - 4\pi k^2 C \delta(\vec{r}). \qquad (16)$$

Originally, such equations where discussed in the framework of the
Bopp-Podolsky electrodynamics. This electrodynamics states that, for $A =
-C$, there exist two kinds of photons, namely massless and massive pho-
tons. They satisfy equations which, in the static spherical-symmetric case,
reduce to Eq. (8). It was shown in [65] that its solution (12) is everywhere
regular and $\phi(0) = ak$ ($a = e$ is the charge of the electron). The same is
true for gravitons: A theory with massless and massive gravitons requires
fourth order equations (see Sec. 3.1).[3]

The atomic hypotheses assuming shielding effects lead in the static case
to a modification of Newton's gravitational law that is approximately given
by the potential introduced by von Seeliger and Majorana. Instead of the
r^2-dependence of the force between the attracting bodies given by the New-
tonian fundamental law,

$$\vec{F}_{12} = -G \int \frac{\rho(\vec{r}_1)\rho(\vec{r}_2)}{r_{12}^3}\vec{r}_{12}d^3x_1d^3x_2 \qquad (17)$$

where G is the Newtonian gravitational constant, one finds,

$$\vec{F}_{12} = -G \int \frac{\rho(\vec{r}_1)\rho(\vec{r}_2)}{r_{12}^3}\vec{r}_{12}e^{(-k\int \rho dr_{12})}d^3x_1d^3x_2. \qquad (18)$$

Here the exponent $-k\int \rho dr_{12}$ means an absorption of the flow of force $\vec{F}$
by the atomic masses between the two gravitating point masses. Since, for
observational reasons, one has to assume that the absorption exponent is
much smaller than 1, as a first approximation, (18) may be replaced by
Laplace's expression,

$$\vec{F}_{12}^* = -G\frac{M_1M_2}{r_{12}^3}\vec{r}_{12} \exp/(-k\rho dr) \approx -G\frac{M_1M_2}{r_{12}^3}\vec{r}_{12}(1 - k\rho\triangle r). \qquad (19)$$

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[3]In [120] it was shown that it is erroneous to assume that massive gravitons occur in
Einsteins GRT with a cosmological term.

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Equation (19) contains a new fundamental constant, namely Majorana's "absorption coefficient of the gravitational flow"

$$k \geq 0, [k] = cm^2 g^{-1}. \tag{20}$$

This value can be tested by the Eötvös experiment, where one can probe whether the ratio of the gravitational and the inertial mass of a body depends on its physical properties. In the case of absorption of gravitation the value of this ratio would depend on the density of the test body. Gravimetric measurements of the gravitational constant carried out by Eötvös by means of a torsion pendulum and gravitational compensators showed that k has to be smaller than

$$k < 4 \times 10^{-13} cm^2 g^{-1}. \tag{21}$$

By comparison Majorana [86, 87, 88] obtained in his first experiments the value

$$k \approx 6.7 \times 10^{-12} cm^2 g^{-1}, \tag{22}$$

which was compatible with his theoretical analysis. (Later, after some corrections, he arrived at about half this value [89]).

A more precise estimation of k can be derived from celestial-mechanical observations. As mentioned above, Bottlinger hypothesized that certain (saros-periodic) fluctuations of the motion of the moon are due to an absorption of solar gravity by Earth when it stands between Sun and Moon[4]. If we assume this hypothesis, then, following Crowley et al. [27], the amplitude λ of these fluctuations is related to the absorption coefficient k via

$$\lambda \approx 2ka\rho, \tag{23}$$

where ρ denotes the mean density and a the radius of the earth. If one assumes that

$$k \approx 6.3 \times 10^{-15} cm^2 g^{-1}, \tag{24}$$

then the value of λ is in accordance with the so-called great empirical term of the moon theory. This, however, also shows that, if the fluctuations of the motion of moon here under consideration had indeed been explained by von Seeliger's absorption hypothesis, then greater values than the one given by (24) are not admissible as they would not be compatible with the motion theory. That there is this celestial-mechanical estimation of an

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[4]A. Einstein commented Bottlinger's theory in [36, 38, 39] (see also [123]).

upper limit for k had already been mentioned by Russell [107] in his critique of Majorana's estimation (22).

A better estimate of k has been reached by measurements of the tidal forces. According to Newton's expression, the tidal force acting upon earth by a mass M at a distance R is

$$Z = -2\frac{GM}{R^2}\frac{a}{R} \tag{25}$$

where a denotes the radius of the earth. However von Seeliger's and Majorana's ansatz (18) provided

$$Z^* \approx -2\frac{GM}{R^2}\frac{a}{R} - \frac{\lambda GM}{2R^2} \approx -2\frac{GMa}{R^3}(1+\frac{\lambda R}{4a}) \tag{26}$$

with the absorption coefficient of the earth body

$$\lambda \approx 2ka\rho \approx 6.6{\times}10^9 cm^{-2}g \times k. \tag{27}$$

Considering now the ratio of the tidal forces due to the sun and moon, Z_s and Z_m, one finds in the Newtonian case

$$\frac{Z_s}{Z_m} \approx \frac{M_s}{M_m}\left(\frac{R_m}{R_s}\right)^3 = \frac{5}{11} \tag{28}$$

and in the von Seeliger-Majorana case

$$\frac{Z_s^*}{Z_m^*} \approx \frac{M_s}{M_m}\left(\frac{R_m}{R_s}\right)^3(1+\lambda\frac{R_s}{a}) = \frac{5}{11}(1 + 4k \times 10^{13}cm^{-2}g). \tag{29}$$

Measurements carried out with a horizontal pendulum by Hecker [60] gave the result

$$\frac{Z_s^*}{Z_m^*} \leq 1, \quad \text{that is,} \quad k < 2{\times}10^{-14}g^{-1}cm^2. \tag{30}$$

This result is still compatible with Bottlinger's absorption coefficient, but not with Majorana's value (22), which provided a sun flood much greater than the moon flood (Russell [107]).

Later, Hecker's estimation was confirmed by Michelson and Gale [91], who by using a "level" obtained

$$\frac{Z_s^*}{Z_m^*} = 0.69 \pm 0.004, \quad \text{i. e.,} \quad k < 1.3{\times}10^{-14}g^{-1}cm^2. \tag{31}$$

(The real precision of these measurements, however, was not quite clear.)

Bottlinger [21, 22] had also proposed to search for jolting anomalies in gravimeter measurements occurring during solar eclipses due to a screening of the gravitational flow of the sun by the moon. In an analysis performed by Slichter et al. [115], however, this effect could not be found and those authors concluded that k has the upper limit $3 \times 10^{-15} cm^2 g^{-1}$. However, as argued earlier [11], measurements of this effect provide by necessity null results due to the equivalence of inertial and passive gravitational masses verified by Eötvös.

The latest observational limits on the size of the absorption coefficient is $k < 10^{-21} cm^2 g^{-1}$. It was established by a reanalysis of lunar laser ranging data (Eckardt [34], cf. also Gillies [56]). This would rule out the existence of this phenomenon, at least in the way that it was originally envisioned. (For an estimation of that part which, from the viewpoint of measurement, is possibly due to shielding effects, cf. [139].) The actual terrestrial experimental limit provides $k < 2 \times 10^{-17} cm^2 g^{-1}$ [130].

About the same estimate follows from astrophysics [116, 123]. Indeed, astrophysical arguments suggest that the value for k has to be much smaller than $10^{-21} cm^2 g^{-1}$. This can be seen by considering objects of large mass and density like neutron stars. In their case the total absorption can no longer be described by the Seeliger-Majorana expression. However, one can utilize a method developed by Dubois-Reymond (see Drude [33]) providing the upper limit $k = 3/R\rho$, where R is the radius and ρ the density of the star. Assuming an object with the radius $10^6 cm$ and a mass equal to $10^{34} g$ one is led to $k = 10^{-22} cm^2 g^{-1}$.

As in the aforementioned experiments, these values for k exclude an absorption of gravitation in accordance with the Seeliger-Majorana model. But it does not rule out absorption effects as described by relativistic theories of gravity like the tetrad theory, where the matter source is coupled potential-like to gravitation [11]. The same is true for other theories of gravity competing with GRT that were systematically investigated as to their experimental consequences in Will [138]. For instance, in the tetrad theory, the relativistic field theory of gravity is constructed such that, in the static non-relativistic limit, one has (for details see Sec 3.2)

$$\triangle \phi = 4\pi G \rho \left(1 - \frac{\alpha |\phi|}{c^2 r_{12}} \right), \quad \alpha = const. \tag{32}$$

From this equation it follows that there is a suppression of gravitation by another mass or by its own mass. In the case of two point masses, the

mutual gravitational interaction is given by

$$m_1 \ddot{\vec{r}}_1 = -\frac{Gm_1 m_2}{r_{12}^3} \vec{r}_{12} \left(1 - \frac{2Gm_1}{c^2 r_{12}}\right),$$

$$m_2 \ddot{\vec{r}}_2 = -\frac{Gm_1 m_2}{r_{12}^3} \vec{r}_{12} \left(1 - \frac{2Gm_2}{c^2 r_{12}}\right). \tag{33}$$

Thus the effective active gravitational mass m is diminished by the suppression factor $(1 - 2Gm/c^2 r_{12})$. Such effects can also be found in gravitational theories with a variable 'gravitational constant' (Dirac [32], Jordan [66, 67], Brans and Dicke [23]). Furthermore, in the case of an extended body one finds a self-absorption effect. The effective active gravitational mass $\bar{M}$ of a body with Newtonian mass M and radius r is diminished by the body's self-field,

$$\bar{M} = GM \left(1 - \frac{4\pi G}{3c^2} \rho r^2\right) \tag{34}$$

where exact calculations show that the upper limit of this mass is approximately given by the quantity $c^2/\sqrt{G\rho}$.

The modifications of the Newtonian law mentioned above result from modifications of the Laplace equation. In their relativistic generalization, these potential equations lead to theories of gravity competing with GRT. On one hand GRT provides, in the non-relativistic static approximation, the Laplace equation and thus the Newtonian potential and, in higher-order approximations, relativistic corrections. On the other hand, the competing relativistic theories lead, in the first-order approximation, to the above mentioned modifications of the Laplace equation and thus, besides the higher-order relativistic corrections, to additional non-Newtonian variations. All these relativistic theories of gravity (including GRT) represent attempts to extend Faraday's principle of the local nature of all interactions to gravitation. Indeed, in GRT the geometrical interpretation of the equivalence principle realizes this principle insofar as it locally reduces gravitation to inertia and identifies it with the local world metric; this metric replaces the non-relativistic potential and Einstein's field equations replace the Poisson's potential equation. Other local theories introduce additional space-time functions which together with the metric describe the gravitational field. In some of the relativistic rivals of GRT, these functions are of a non-geometric nature. An example is the Jordan or Brans-Dicke scalar field, which, in accordance with Dirac's hypothesis, can be interpreted as a variable gravitational 'constant' G. (For a review of theories involving absorption and suppression of gravitation, see [11].) In other theories of gravity, these additional functions are essentials of the geometric framework, as

for instance tetrad theories working in teleparallel Riemannian space and the metric-affine theories working in Riemann-Cartan space-times that are characterized by non-vanishing curvature and torsion (see Secs 3.2, 3.3).

3. On absorption, self-absorption and suppression of gravitation in relativistic theories

The original idea of shielding gravitation assumed that the insertion of some kind of matter between the source of gravitation and a test body could reduce the gravitational interaction. This would be a genuine non-Einsteinian effect. However, in the present paper, our main concern will be relativistic theories. Those theories satisfy the Einstein Equivalence Principle (EEP) [11, 138] stating the following:

  i) The trajectory of an uncharged test body depends only on the initial conditions, but not on its internal structure and composition, and
  ii) the outcome of any non-gravitational experiment is independent of the velocity of the freely falling apparatus and of the space-time position at which it is performed (local Lorentz invariance and local position invariance).

The point is that, as long as one confines oneself to theories presupposing the EEP, shielding effects will not occur. For, this principle implies that there is only one sign for the gravitational charge such that one finds quite another situation as in electrodynamics where shielding effects appear ("Faraday screen"). But other empirical possibilities for (non-GRT) relativistic effects like absorption and self-absorption are not excluded by the EEP.

In the case of absorption the intervening matter would not only effect on the test body by its own gravitational field but also by influencing the gravitational field of the source. If the latter field is weakened one can speak of absorption. (For a discussion of this effect, see also [129]). In contrast to absorption, self-absorption describes the backreaction of the gravitational field on its own source so that the effective active gravitational mass of a body (or a system of bodies) is smaller than its uneffected active gravitational mass. Of course, both absorption and self-absorption effects can occur simultaneously, and generally both violate the Strong Equivalence Principle (SEP) [11, 138] stating:

  i) The trajectories of uncharged test bodies as well as of self-gravitating bodies depend only on the initial conditions, but not on their internal structure and composition, and
  ii) the outcome of any local test experiment is independent of the velocity of the freely falling apparatus and of the space-time position at which it is performed.

GRT fulfils SEP. The same is true for fourth-order metric theories of gravity, although there one has a suppression (or amplification) of the matter source by its own gravitational field (see Sec. 3.1).

To empirically test gravitational theories, in particular GRT, it is helpful to compare them to other gravitational theories and their empirical predictions. For this purpose, it was most useful to have a general parameterized framework encompassing a wide class of gravitational theories where the parameters, whose different values characterize different theories, can be tested by experiments and observations. The most successful scheme of this type is the Parameterized Post-Newtonian (PPN) formalism [138] confronting the class of metric theories of gravity with solar system tests.

This formalism rendered it possible to calculate effects based on a possible difference between the inertial, passive and active gravitational masses. It particularly was shown that some theories of gravity predict a violation of the first requirement of the SEP that is due to a violation of the equivalence between inertial and passive gravitational masses. This effect was predicted by Dicke [31] and calculated by Nordtvedt [97]. It does not violate the EEP. Other effects result from a possible violation of the equivalence of passive and active gravitational masses. For instance, Cavendish experiments could show that the locally measured gravitational constant changes in dependence on the position of the measurement apparatus. This can also be interpreted as a position-dependent active gravitational mass, i.e., as absorption.

The PPN formalism is based on the four assumptions:

 i) slow motion of the considered matter,
 ii) weak gravitational fields,
iii) perfect fluid as matter source, and
iv) gravitation is described by so-called metric theories (for the definition of metric theories, see below).

As noticed in [138], due to the first three assumptions, this approach is not adequate to compare the class of gravitational theories described in (iv) with respect to compact objects and cosmology. And, of course, in virtue of assumption (iv), the efficiency of this approach is limited by the fact that it is dealing with metric theories satisfying EEP.

During the last decades great efforts were made to generalize this framework to nonmetric gravitational theories and to reach also a theory-independent parameterization of effects which have no counterpart in the classical domain. (For a survey, see e. g. [73, 57].) The generalization to a nonmetric framework enables one to test the validity of the EEP, too, and the analysis of "genuine" quantum effects leads to new test possibilities in gravitational physics, where these effects are especially appropriate to test the coupling of quantum matter to gravitation.

First of all, this generalized framework is appropriate to analyze terrestrial, solar system, and certain astrophysical experiments and observations. Of course, it would be desirable to have also a general framework for a systematic study of gravitational systems like compact objects or cosmological models in alternative theories of gravity. But one does not have it so that one is forced to confine oneself to case studies what, to discuss relativistic absorption effects, in the next section will be done.

Our considerations focus on theories that use the genuinely geometrical structures metric, tetrads, Weitzenböck torsion, and Cartan torsion for describing the gravitational field. Accordingly, the following case studies start with theories formulated in a Riemannian space, go then over to theories in a teleparallelized space, which is followed by a simple example of a theory established in a Riemann-Cartan space[5].

But before turning to this consideration, a remark concerning the relation between EEP and metric theories of gravity should be made. This remark is motivated by the fact that it is often argued that the EEP necessarily leads to the class of metric theories satisfying the following postulates:

 i) Spacetime is endowed with a metric **g**,
 ii) the world lines of test bodies are geodesics of that metric, and
 iii) in local freely falling frames, called local Lorentz frames, the non-gravitational physics are those of special relativity.

(Therefore, presupposing the validity of EEP, in [138] only such theories are compared.)

The arguments in favor of this thesis are mainly based on the observation (see, e.g., [138], p.22 f.) that one of the aspects of EEP, namely the local Lorentz invariance of non-gravitational physics in a freely falling local frame, demands that there exist one or more second-rank tensor fields which reduce in that frame to fields that are proportional to the Minkowski met-

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[5]Of course, there are other alternatives to GRT which are also interesting to be considered with respect to non-Einsteinian effects, such as scalar-tensor theories of Brans-Dicke type and effective scalar-tensor theories resulting from higher-dimensional theories of gravity by projection, compactification or other procedures (for such theories, see also the corresponding contributions in this volume). At first the latter idea was considered by Einstein and Pauli [44, 52]. They showed that, if a Riemannian V_5 with signature (-3) possesses a Killing vector which is orthogonal to the hypersurface $x^4 = const$, V_5 reduces to a V_4 with a scalar field because then one has $(A, B = 0, \ldots, 4)$: $g_{AB,4} = 0, g_{i4} = 0, g_{44} = (g^{44})^{-1} = g_{44}(x^l)$.
It should be mentioned that the fourth-order theories discussed in Sec. 3.1 and the scalar-tensor theories are interrelated (cf., the references to equivalence theorems given in Sec.3.1). Furthermore, there is also an interrelation between fourth-order theories, scalar-tensor theories, and theories based on metric-affine geometry which generalizes Riemann-Cartan geometry by admitting a non-vanishing non-metricity. First this became obvious in Weyl's theory [133] extending the notion of general relativity by the requirement of conformal invariance and in Bach's interpretation of Weyl's "unified" theory as a scalar-tensor theory of gravity [3] (see also, [17]).

ric η. Another aspect of EEP, local position invariance then shows that the scalar factor of the Minkowski metric is a universal constant from which the existence of a unique, symmetric, second-rank tensor field **g** follows. However, the point we want to make here is that this does not automatically mean that the gravitational equations have to be differential equations for the metric (and, possibly, additional fields, as for instance a scalar field which do not couple to gravitation directly, as, e.g. realized in the Brans-Dicke theory [23]). This conclusion can only be drawn if one assumes that the metric is a primary quantity. Of course, in physically interpretable theory, one needs a metric and from EEP it follows the above-said. But one can also consider gravitational theories, wherein other quantities are primary. Then one has field equations for these potentials, while the metric describing, in accordance with EEP, the influence of gravitation on nongravitational matter, is a secondary quantity somehow derived from the primary potentials. For instance, such theories are tetrad (or teleparallelized) theories [93, 94, 101] (see also [11, 12, 15]) or affine theories of gravity [41, 42, 43, 50, 110] (see also [16, 17]).

3.1. FOURTH-ORDER THEORIES

Fourth-order derivative equations of gravitation have received great attention since several authors have proved that the fourth-order terms R^2, $R_{ik}R^{ik}$, and $R_{iklm}R^{iklm}$ can be introduced as counter-terms of the Einstein-Hilbert Lagrangian R to make GRT, quantized in the framework of covariant perturbation theory, one-loop renormalizable. In this context it was clarified that higher-derivative terms naturally appear in the quantum effective action describing the vacuum polarization of the gravitational field by gravitons and other particles [29, 30, 28, 25]. Such equations revive early suggestions by Bach [3], Weyl [134, 135], Einstein [40], and Eddington [35] (see also Pauli [99] and Lanczos [74, 75]).

While early papers treated such equations formulated in non-Riemannian spacetime in order to unify gravitational and electromagnetic fields[6], later they were also considered as classical theories. From the viewpoint of classical gravitational equations with phenomenological matter modifying the Einstein gravitation at small distances, the discussion of such field equations was opened by Buchdahl [24] and Pechlaner and Sexl [100]. Later this discussion was anew stimulated by the argument [121, 124] that such equations should be considered in analogy to the fourth-order electromagnetic theory of Bopp and Podolsky [20, 102] in order to solve the singularity and collapse problems of GRT. Especially, it was shown [121, 124]

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[6]Higher-derivative equations were also studied within the framework of quantum field theory by Pais and Uhlenbeck [98].

that, for physical reasons, the Einstein-Hilbert part of the Lagrangian must be necessarily included (for this point, see also [58]) and that one has to impose supplementary conditions on the matter source term. These considerations were continued in [18, 19] (see also [8, 9, 10]). The consequences for singularities of massive bodies and cosmological collapse were also analyzed in [117, 54]. (In the following, we shall call fourth-order equations supplemented by the second-order Einstein term *mixed fourth-order equations.*)

The meaning of such higher order terms in early cosmology was considered in [5, 78]. Especially the influence of pure R^2-terms in cosmological scenarios can be interpreted as a pure gravitational source for the inflationary process [92, 6, 84]. Moreover it was shown in [137] that the transformation $\tilde{g}_{ab} = (1 + 2\epsilon R)g_{ab}$ leads back to GRT described by $\tilde{g}_{ab}$ with an additional scalar field R acting as a source. (For more general equivalence theorems, see [85] and the corresponding references cited therein.)

In the case of stellar objects in the linearized, static, weak field limit for perfect fluid the corresponding Lane-Emden equation provides a relative change of the radius of the object compared to Newtons theory depending on the value of the fundamental coupling constants [26]. This may decrease or increase the size of the object.

Mixed fourth-order equations may only be considered as gravitational field equations, if they furnish approximately at least the Newton-Einstein vacuum for large distances. To this end, it is not generally sufficient to demand that the static spherical-symmetric solutions contain the Schwarzschild solution the Newtonian potential in the linear approximation. One has to demand that the corresponding exact solution can be fitted to an interior equation for physically significant equations of state. In the case of the linearized field equations, this reduces to the requirement that there exist solutions with suitable boundary conditions which, for point-like particles, satisfy a generalized potential equation with possessing a delta-like source.

The most general Lagrangian containing the Hilbert-Einstein invariant as well as the quadratic scalars reads as follows[7] (the square of the curvature tensor need not be regarded since it can be expressed by the two other terms, up to a divergence):

$$L = \sqrt{-g}R + \sqrt{-g}(\alpha R_{ab}R^{ab} + \beta R^2)l^2 + 2\kappa L_M \tag{35}$$

where α and β are numerical constants, l is a constant having the dimension of length. Variation of the action integral $I = \int L d^4x$ results in the field

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[7]In this section, we mainly follow [18, 19].

equations of the fourth order, viz.,

$$l^2 H_{ab} + E_{ab} := \quad (36)$$

$$= l^2 \big(\alpha \Box R_{ab} + (\frac{\alpha}{2} + 2\beta) g_{ab} \Box R - (\alpha + 2\beta) R_{;ab} + 2\beta R R_{ab} -$$

$$-\frac{\beta}{2} g_{ab} R^2 + 2\alpha R_{acdb} R^{cd} - \frac{\alpha}{2} g_{ab} R_{cd} R^{cd} \big) + \left(R_{ab} - \frac{1}{2} R g_{ab} \right) = -\kappa T_{ab}.$$

(Here $\Box$ denotes the covariant wave operator.) These equations consist of two parts, namely, the fourth-order terms $\propto l^2$ stemming from the above quadratic scalars and the usual Einstein tensor, where

$$T_{ab} := \frac{1}{\sqrt{-g}} \frac{\delta L_M}{\delta g^{ab}} \quad (\text{with} \quad T^a{}_{b;a} = 0). \tag{37}$$

For $\alpha = -2\beta = 1$ (assumed by Eddington in the case of pure fourth-order equations), in the linear approximation the vacuum equations corresponding to Eqs. (36) reduce to

$$l^2 \Box_\eta E^1_{ab} + E^1_{ab} = 0 \tag{38}$$

and, for $\alpha = -3\beta = 1$ (assumed by Bach and Weyl in the case of pure fourth-order equations), to

$$l^2 \Box_\eta R^1_{ab} + R^1_{ab} = 0 \tag{39}$$

(the latter follows because in this case the trace of the vacuum equations provides $R = 0$). The discussion of these equations performed in [18] shows that both cases are characterized in that they possess, besides massless gravitons, only *one* kind of gravitons with non-vanishing restmass. (These results were corroborated by subsequent considerations [8, 10].)

In the linear approximation for static fields with the Hilbert coordinate condition

$$g^{ik}{}_{,k} = \frac{1}{2} g^k{}_{k,i} \tag{40}$$

in the above-considered two cases, the pure and mixed fourth order field equations rewritten in a compact form as

$$l^2 H_{ik} = -\kappa T_{ik}, \tag{41}$$

$$l^2 H_{ik} + E_{ik} = -\kappa T_{ik}, \tag{42}$$

produce the simple potential equations (9) and (10) for all components of the metric tensor, if $\alpha = -3\beta$ or $\alpha = -2\beta$, where $k \propto l^{-1}$ is a reciprocal

length, and a is the mass of a point-like particle which is given by the Dirac delta-function δ. As shown in Sec. 2, it is this condition imposed on the source that leads to the Green functions (11) and (12). From the expressions (11) and (13) it is evident that, in the case of pure field equations of fourth order, this condition will reduce the manifold of solutions to a functions that do not satisfy the correct boundary conditions at large distances, i.e., those field equations do not mirror the long-range Newtonian interaction. They are consequently at most field equations describing free fields, in other words, equations of a unified field theory in the sense of Weyl's [134] and Eddington's [35] ansatz.

The requirement of general coordinate covariance furnishes in the fourth-order case a variety of field equations depending upon the parameters α and β, whereas the Einstein equations are determined by this symmetry group up to the cosmological term. If one postulates that the field equations are invariant with respect to conform transformations, one obtains just the pure fourth-order equations of Bach and Weyl, where $\alpha = -3\beta$. However, if one wants to couple gravitation to matter one has to go a step further since, due to the conform invariance, the trace of H_{ik} vanishes. Accordingly, the trace of the matter must also vanish. That means that in this case there are supportive reasons for the replacement of pure by mixed field equations. Then the conform invariance is broken by the term E_{ik} being the source of massless gravitons [18].

The meaning of the short-range part of the gravitational potential of this theory becomes more evident when one has a glance at the stabilizing effect it has in the classical field-theoretical particle model (for more details, see [19]). Within GRT, such models first were tried to construct by Einstein [37], Einstein and Rosen [53], and Einstein and Pauli [52]. Later this work was continued in the frame of the so-called geon program by Wheeler and co-workers. Already in the first paper by Einstein dealing with this program it was clear that, in the Einstein-Maxwell theory, it was difficult to reach a stable model. In particular, it was shown that a stable model requires to assume a cut-off length and the equality of mass and charge. The fact that the situation in regard to the particle problem becomes more tractable under these two conditions is an indication that GRT should be modified, if one wants to realize this program. As far as very dense stars are concerned, such a modification should already become significant if the distances are of the order of magnitude of the gravitational radii.

Indeed, such a modification is given by the mixed field equations. In the linear approximation given by Eq. (10), one is led to the Podolsky type solution (Greek indices run from $1 \ldots 3$),

$$g_{00} = 1 + 2\phi, \quad g_{0\alpha} = 0, \quad g_{\alpha\beta} = -\delta_{\alpha\beta}(1 - 2\phi) \tag{43}$$

where

$$\phi = \frac{GM}{rc^2}(1 - e^{-kr}).$$ (44)

By calculating the affine energy-momentum tensor $t^i{}_k$ of the gravitational field with this solution and integrating $t^i{}_k$ over a spatial volume, one obtains the result that the Laue criterion [77] for a stable particle with finite mass yields:

$$\int t^0{}_0 d^3x = \frac{GM^2}{2}c^2k, \quad \int t^0{}_\alpha d^3x = \int t^\alpha{}_0 d^3x = 0.$$ (45)

This character of the Podolsky potential can also be seen if one introduces it in the Einstein model [49] of a stellar object (see [7]).

The left-hand pure vacuum part of the field equations (36) with arbitrary α and β are constructed according to the same scheme as Einstein's second-order equations. They are determined by the requirement that they are not to contain derivatives higher than fourth order and fulfil the differential identity $H^k{}_{i;k} = 0$. By virtue of this identity and the contracted Bianchi identity for E_{ik}, $E^k{}_{i;k} = 0$, we get, as in the case of Einstein's equations, the dynamic equations

$$T^k{}_{i;k} = 0,$$ (46)

for the pure and mixed equations of fourth order. Consequently, the EEP is also satisfied for those equations.

As to the SEP, the fourth-order metric theory satisfies all conditions of this principle formulated above. But this does not exclude a certain back-reaction of the gravitational field on its matter source leading to the fact that the effective active gravitational mass differs from the inertial mass. This can made plausible by the following argument [9]: Regarding that for vacuum solutions of Eqs. (36) satisfying the asymptotic condition

$$g_{00} = 1 - \frac{2a}{r} \quad \text{for} \quad r \to \infty$$ (47)

the active gravitational mass is given by (see [52])

$$a = \int \int \int (E^0{}_0 - E^\alpha{}_\alpha)\sqrt{-g}d^3x$$ (48)

we obtain

$$a = \kappa \int \int \int (T^0{}_0 - T^\alpha{}_\alpha)\sqrt{-g}d^3x - l^2 \int \int \int (H^0{}_0 - H^\alpha{}_\alpha)\sqrt{-g}d^3x.$$ (49)

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This relation replaces the GRT equation

$$a = \kappa \int \int \int (T^0{}_0 - T^\alpha{}_\alpha)\sqrt{-g}d^3x. \qquad (50)$$

It represents a modification of the Einstein-Newtonian equivalence of active gravitational mass a and inertial mass m stemming from the hidden-matter term $\propto l^2$. It behaves like hidden (or "dark") matter since, due to the relation $H^{ik}{}_{;k} = 0$, H^{ik} decouples from the visible-matter term T^{ik}. (In the case $\alpha = -3\beta$, this term may be interpreted as a second kind of gravitons). In dependence on the sign of this term, it suppresses or amplifies the active gravitational mass.

3.2. TETRAD THEORIES

That class of gravitational theories which leads to a potential-like coupling is given by tetrad theories formulated in Riemannian space-time with teleparallelism. To unify electromagnetism and gravitation it was introduced by Einstein and elaborated in a series of papers, partly in cooperation with Mayer (for the first papers of this series, see [45, 48, 46, 47, 51]). From another standpoint, later this idea was revived by Møller [93, 94] and Pellegrini & Plebański [101], where the latter constructed a general Lagrangian based on Weitzenböck's invariants [81]. These authors regard all 16 components of the tetrad field as gravitational potentials which are to be determined by corresponding field equations. The presence of a non-trivial tetrad field can be used to construct, beside the Levi-Civita connection defining a Riemannian structure with non-vanishing curvature and vanishing torsion, a teleparallel connection with vanishing curvature and non-vanishing torsion. This enables one to build the Weitzenböck invariants usable as Lagrangians. (Among them, there is also the tetrad equivalent of Einstein-Hilbert Lagrangian, as was shown by Møller.)

A tetrad theory of gravity has the advantage to provide a satisfactory energy-momentum complex [93, 101]. Later Møller [94] considered a version of this theory that can free macroscopic matter configurations of singularities.

From the gauge point of view, in [63] tetrad theory was regarded as translational limit of the Poincaré gauge field theory, and in [95] such a theory was presented as a constrained Poincaré gauge field theory[8]. Another approach [59] considers the translational part of the Poincaré group as gauge group, where in contrast to Poincaré gauge field theory this theory is assumed to be valid on microscopic scales, too. A choice of the Lagrangian leading to a more predictable behavior of torsion than in the

[8]For Poincaré gauge field theory, see Sec. 3.3.

above-mentioned versions of the tetrad theory is discussed in [70] - from the point of Mach's principle, tetrad theory was considered in [12].

The general Lagrange density which is invariant under global Lorentz transformations and provides differential equations of second order is given by

$$L^* = \sqrt{-g}(R + aF_{Bik}F^{Bik} + b\Phi_A\Phi^A) + 2\kappa L_M \qquad (51)$$

where a, b are constants, $R = g^{ik}R_{ik}$ is the Ricci scalar, L_M denotes the matter Lagrange density and

$$F_{Aik} := h_{Ak,i} - h_{Ai,k} = h_A{}^l(\gamma_{lki} - \gamma_{lik}), \quad \Phi_A := h_A{}^i\gamma^m{}_{im}. \qquad (52)$$

Here $h_A{}^l$ denote the tetrad field and $\gamma_{lki} = h^A{}_l h_{Ak;i}$ the Ricci rotation coefficients ($A, B, \ldots$ are tetrad (anholonomic) indices, $i, k, \ldots$ are space-time (holonomic) indices).

To consider absorption and self-absorption mechanisms we confine ourselves to the case $b = 0$, such that the Lagrange density takes the form

$$L = \sqrt{-g}(R + aF_{Bik}F^{Bik}) + 2\kappa L_M. \qquad (53)$$

(Einstein [45] and Levi-Civita [81] discussed the corresponding vacuum solution as a candidate for a unified gravito-electrodynamic theory.) Introducing the tensors

$$T_{ik} := \frac{1}{\sqrt{-g}}\frac{\delta L_M}{\delta h^i{}_A}h^A{}_k \qquad (54)$$

$$G_{ik} := R_{ik} - \frac{1}{2}g_{ik}R + \kappa T_{ik} + 2a\left(\frac{1}{4}g_{ik}F_{Bmn}F^{Bmn} - F_{Bim}F^B{}_k{}^m\right) \qquad (55)$$

and the 4-vector densities

$$S_A{}^i = \sqrt{-g}h_A{}^k G^i{}_k \qquad (56)$$

and varying the Lagrangian (53) with respect to the tetrad field, it follow the gravitational field equations in the "Maxwell" form

$$F_A{}^{ik}{}_{;k} = \frac{1}{2a}S_A{}^i. \qquad (57)$$

These equations can also be rewritten in an "Einstein" form. This form stems from a Lagrangian which differs from (53) by a divergence term [126]. Since, in the following paragraphs of this section, we assume a symmetric energy-momentum tensor this form be given for this special case:

$$R_{ik} - \frac{1}{2}g_{ik}R = -\kappa T_{ik} - \Theta_{(ik)} \qquad (58)$$

$$\Theta_{ik} - \Theta_{ki} = 0 \qquad (59)$$

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where

$$\Theta_{ik} = a(\frac{1}{2}g_{ik}F_{Bmn}F^{Bmn} - F_{Bim}F^B{}_k{}^m + 2h^A{}_i F_{Ak}{}^l{}_{;l}).$$ (60)

The latter form of the field equations allows for an interesting interpretation, because the purely geometric term (60) can be regarded as matter source term[9]. Furthermore, for a symmetric tensor T_{ik}, due to the dynamical equations and Bianchi's identities, one has [12]

$$\Theta_i{}^k{}_{;k} = 0$$ (61)

so that Θ_{ik} does not couple to visible matter described by T_{ik}. Again it behaves like hidden (or "dark") matter.

The Maxwell form (57) shows clearly that this theory can be considered as a general-relativistic generalization of Eq. (3). Interestingly, in this form the gravitational (suppressing or amplifying) effect appears as a combination of an absorption effect given by the potential-like coupling of usual matter and a hidden-matter effect. Of course, both versions of the theory must lead to the same empirical results.

To discuss absorption effects following from the potential-like coupling in more detail, let us consider the absorption of the active gravitational mass of Earth by the gravitational field of Sun, i.e., calculate the change of the spherical-symmetric part of the Earth field in dependence on the position in the Sun field. To do this, we shall follow a method used in [82] for equations similar to (57) (see also [11]). This effect gives an impression of the order of magnitude of the effects here under consideration.

To this end, we consider the field of Earth, $h^A_{\underset{1}{i}}$, as a perturbation of the field of Sun, $h^A_{\underset{0}{i}}$:

$$h^A{}_i = h^A_{\underset{0}{i}} + h^A_{\underset{1}{i}} \quad \text{with} \quad |h^A_{\underset{1}{i}}| \ll |h^A_{\underset{0}{i}}|.$$ (62)

Rewriting Eq. (57) in the form

$$\Box h^A{}_i - h^{Ak}{}_{,ik} = \frac{h_{,k}}{h}F^A{}_i{}^k + \frac{1}{2a}h^A{}_l E^l{}_i + \frac{\kappa}{2a}h^A{}_l T^l{}_i +$$ (63)
$$+ \frac{1}{4}h^A{}_i F_{Bmn}F^{Bmn} - h^{Al}F_{Blm}F^B{}_i{}^m$$

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[9]If one considers the vacuum version of (58) in the context of unified geometric field theory this interpretation is even forcing (see Schrödinger [110]).

(where $h := \sqrt{-g} = det|h^A{}_i|$ and $E_{ik} = R_{ik} - \frac{1}{2}g_{ik}R$) and inserting ansatz (62) one obtains in the first order approximation

$$\Box h^A_{\underset{0}{i}} + \Box h^A_{\underset{1}{i}} - h^{Ak}_{\underset{0}{,ik}} - h^{Ak}_{\underset{1}{,ik}} = \frac{h_{,k}}{\underset{0}{h}}F^A_{\underset{0}{i}}{}^{k} + \frac{h_{,k}}{\underset{0}{h}}\underset{1}{h}F^A_{\underset{0}{i}}{}^{k} + h_{,k}F^A_{\underset{1}{0}}{}^{k} + \quad (64)$$

$$+\frac{h_{,k}}{\underset{0}{h}}F^A_{\underset{1}{i}}{}^{k} + \frac{1}{2a}h^A_{\underset{0}{l}}E^l_{\underset{1}{i}} + \frac{1}{2a}h^A_{\underset{1}{l}}E^l_{\underset{0}{i}} + \frac{1}{2a}h^A_{\underset{0}{l}}E^l_{\underset{0}{i}} + \frac{1}{4}h^A_{\underset{0}{l}}F_{Bmn}F^{Bmn}_{\underset{0}{}} +$$

$$+\frac{1}{4}(h^A{}_i F_{Bmn}F^{Bmn})_1 - h^{Al}_{\underset{0}{}}F_{Blm}F^B_{\underset{0}{i}}{}^m - (h^{Al}F_{Blm}F^B{}_i{}^m)_1 +$$

$$+\frac{\kappa}{2a}\left(h^A_{\underset{0}{l}}\underset{E}{T}^l_{i} + h^A_{\underset{1}{l}}\underset{S}{T}^l_{i} + h^A_{\underset{1}{l}}\underset{E}{T}^l_{i} + h^A_{\underset{0}{l}}\underset{S}{T}^l_{i}\right),$$

where $\underset{E}{T}^l{}_i$ and $\underset{S}{T}^l{}_i$ denote the energy-momentum tensor of Earth and Sun, respectively, and $\sqrt{-g} := h = \underset{0}{h} + \underset{1}{h}$. Regarding that the solar field $h^A_{\underset{0}{i}}$ is a solution of that equation which is given by the zero-order terms in Eq. (64) one gets as first-order equation for $h^A_{\underset{1}{i}}$

$$\Box h^A_{\underset{1}{i}} - h^{Ak}_{\underset{1}{,ik}} = \frac{\kappa}{2a}\left(h^A_{\underset{0}{l}}\underset{E}{T}^l_{i} + h^A_{\underset{1}{l}}\underset{S}{T}^l_{i} + h^A_{\underset{1}{l}}\underset{E}{T}^l_{i}\right) + \quad (65)$$

$$+\frac{h_{,k}}{\underset{0}{h}}\underset{1}{h}F^A_{\underset{0}{i}}{}^{k} + h_{,k}F^A_{\underset{1}{0}}{}^{k} + \frac{h_{,k}}{\underset{0}{h}}F^A_{\underset{1}{i}}{}^{k} + \frac{1}{2a}h^A_{\underset{0}{l}}E^l_{\underset{1}{i}} +$$

$$+\frac{1}{2a}h^A_{\underset{1}{l}}E^l_{\underset{0}{i}} + \frac{1}{4}(h^A{}_i F_{Bmn}F^{Bmn})_1 - (h^{Al}F_{Blm}F^B{}_i{}^m)_1.$$

Now one can make the following assumptions

i) The terms in the second and third lines all contain cross-terms in $h^A_{\underset{0}{l}}$ and $h^A_{\underset{1}{l}}$. They describe, in the first-order approximation, the above-mentioned hidden matter correction of the active gravitational mass. In the following we assume that the constant a is so small that the usual matter terms in the first line of (65) are dominating.

ii) The term $h^A_{\underset{1}{l}}\underset{S}{T}^l_{i}$ is neglected. It describes the influence of Earth potential on the source of the solar field. Near the Earth, it causes a correction having no spherical-symmetric component with respect to the Earth field.

iii) The energy-momentum tensor is assumed to have the form $\underset{E}{T}^{ik} = \rho u^i u^k$ (with $\rho = const$). A more complicated ansatz regarding the internal structure of the Earth in more detail leads to higher-order corrections.

iv) The term $h^A_1{}_l T^l_E{}_i$ is neglected, too. It describes the influence of the Earth field on its own source leading to higher-order (self-absorption) effects.

As a consequence of these assumptions, Eqs. (65) reduce to the field equations

$$\Box h^A_1{}_i - h^{Ak}_1{}_{,ik} = \frac{\kappa}{2a} h^A_0{}_l T^l_E{}_i \qquad (66)$$

Assuming coordinates, where the spherical-symmetric solution $h^A_1{}_i$ has the form ($\mu, \nu = 1, 2, 3$),

$$h^{\hat{0}}_1{}_0 = \alpha, \quad h^{\hat{\mu}}_1{}_\nu = \delta^{\hat{\mu}}_\nu \beta, \quad h^{\hat{0}}_1{}_\nu = h^{\hat{\nu}}_1{}_0 = 0, \qquad (67)$$

Eqs. (66) lead to the equations

$$\triangle \alpha = -\frac{\kappa}{2a} h^0_0{}_l T^l{}_0 \qquad (68)$$

$$\triangle \beta = -\frac{\kappa}{4a} h^\mu_0{}_\nu T^\nu{}_\mu.$$

Therefore, up to a factor, the calculation of the absorption effect provides the same result as given in [125]. Thus a gravimeter would register an annual period in the active gravitational mass of the Earth depending on the distance Earth-Sun. The mass difference measured by a gravimeter at aphelion and perihelion results as

$$\frac{\triangle M}{M} = \frac{4GM_\odot}{c^2 R} \epsilon \frac{1}{2a} \qquad (69)$$

($M_\odot = $ is the mass of the Sun, $R = $ the average distance of the planet from the Sun, and $\epsilon = $ the eccentricity of the planetary orbit). For Earth, this provides the value 6.6×10^{-10} and, for Mercury 2.06×10^{-8} if $2a = 1$.

It should be mentioned that the above-made estimation is performed under the assumption that the planet rests in the solar field (what enables one to assume (67)). However, one has to regard that the square of the velocity of the Earth with respect to the Sun is of the same order of magnitude as the gravitational field of the Sun. Thus, the velocity effect could compensate the absorption effect. As was shown in [82], the velocity effect is of the same order of magnitude, but generally it differs from the absorption effect by a factor of the order of magnitude 1.

3.3. EINSTEIN-CARTAN-KIBBLE-SCIAMA THEORY

The Einstein-Cartan-Kibble-Sciama theory [111, 112, 68] (see also, for an early ansatz [131] and for a later detailed elaboration [61]) is the simplest

example of a Poincaré gauge field theory of gravity, aiming at a quantum theory of gravity. Moreover, the transition to gravitational theories that are formulated in Riemann-Cartan space is mainly motivated by the fact that then the spin-density of matter (or spin current) can couple to the post-Riemannian structure.[10] Therefore, the full content of this theory becomes only obvious when one considers the coupling of gravitation to spinorial matter.

As far as the non-quantized version of the theory is concerned, there exist two elaborated descriptions of spinning matter in such a theory, the classical Weyssenhoff model [136, 71] and a classical approximation of the second quantized Dirac equation [4]. Moreover there is a number of interesting cosmological solutions which show that in such a theory it is possible to prevent the cosmological singularity [69, 72]. But, unfortunately this is not a general property of Einstein-Cartan-Kibble-Sciama cosmological solutions [71].

In the following we shall describe the situation of an effective Einstein-Cartan-Kibble-Sciama theory with Dirac-matter as source term. This theory is formulated in the Riemann-Cartan space U_4 [109], where the non-metricity Q_{ijk} is vanishing, $Q_{ijk} := -\nabla_i g_{jk} = 0$, such that the connection is given as:

$$\Gamma_{ij}^k = \left\{ \begin{matrix} k \\ ij \end{matrix} \right\} - K_{ij}{}^k \tag{70}$$

with the Christoffel symbols $\left\{ \begin{matrix} k \\ ij \end{matrix} \right\}$, the torsion $S_{ij}{}^k$, and the contorsion $K_{ij}{}^k$, which in the holonomic representation read,

$$\left\{ \begin{matrix} k \\ ij \end{matrix} \right\} := \frac{1}{2} g^{kl} (g_{li,j} + g_{jl,i} - g_{ij,l}), \tag{71}$$

$$S_{ij}{}^k := \frac{1}{2} (\Gamma_{ij}^k - \Gamma_{ji}^k), \quad K_{ij}{}^k := -S_{ij}{}^k + S_j{}^k{}_i - S^k{}_{ij} = -K_i{}^k{}_j.$$

In the anholonomic representation, where all quantities are referred to an orthonormal tetrad of vectors $\overrightarrow{e}_A = e^i_A \partial_i$ (capital and small Latin indices run again from 0 to 3)[11], the pure metric part of the anholonomic connection $\tilde{\Gamma}$ is given by the Ricci rotation coefficients $\gamma_{AB}{}^i$.

The field equations are deduced by varying the action integral corresponding to a Lagrange density that consists of a pure gravitational and a

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[10]For modern motivation and representation of metric-affine theories of gravity, see [62].

[11]In contrast to Sec. 3.2, we denote the tetrad with e instead of h since here they are coordinates in the Riemann-Cartan space but not a fixed tetrad field in Riemann space with teleparallelism.

matter part; in the holonomic version one has

$$L = L_G(g, \partial g, \Gamma, \partial \Gamma) + 2\kappa L_M(g, \partial g, \Gamma, \phi, \partial \phi) \qquad (72)$$

and, in the anholonomic version,

$$L = L_G(e, \partial e, \tilde{\Gamma}, \partial \tilde{\Gamma}) + 2\kappa L_M(e, \partial e, \tilde{\Gamma}, \phi, \partial \phi). \qquad (73)$$

In the holonomic representation, one has to vary (72) with respect to

$$g_{ij} \text{ and } \Gamma_{ij}^k \text{ or, for } \Gamma = \Gamma(K, g), \ g_{ij} \text{ and } K_{ij}{}^k$$
$$\text{or, for } K = K(S, g), \ g_{ij} \text{ and } S_{ij}{}^k$$

and, in the anholonomic representation, (73) with respect to

$$e_A^i \text{ and } \tilde{\Gamma}_{AB}{}^i \text{ or, for } \tilde{\Gamma} = \tilde{\Gamma}(K, \gamma), \ e_A^i \text{ and } K_{AB}{}^i.$$

If matter with half-integer spin is to be coupled to gravitation, then one has to transit to the spinorial version of the latter, where one has to vary

$$\gamma^i = e_A^i \gamma^A \text{ and } \omega_{AB}{}^i \text{ (or } S_{AB}{}^i)$$

(γ^A denote the Dirac matrices defined by $\{\gamma^A, \gamma^B\} = 2\eta^{AB}$ and $\omega_{AB}{}^i$ the spinor connection and $S_{AB}{}^i$ the spinor torsion).

In the case of the Kibble-Sciama theory the pure gravitational Lagrangian is assumed to be the Ricci scalar such that $L_G = eR$ (with $e := \det(e_i^A)$, e_i^A is dual to e_A^i). Then the variation of (73) by e_A^i and ω_{iAB} provides the gravitational equations[12]:

$$R^A{}_i - \frac{1}{2} e^A{}_i R = -\kappa T^A{}_i, \qquad (74)$$

$$T_{ikl} = -\frac{\kappa}{2} s_{ikl} \qquad (75)$$

where

$$R^A{}_i - \frac{1}{2} e^A{}_i R := e^k{}_B R_{ik}{}^{AB} - \frac{1}{2} e^A{}_i e^l{}_C e^m{}_D R_{lm}{}^{CD} \qquad (76)$$

$$T_{ikl} := S_{ikl} - 2\delta^l_{[i} S_{k]m}{}^m, \qquad (77)$$

$$T_i{}^A := -\frac{1}{e} \frac{\delta L_M}{\delta e^i{}_A}, \qquad (78)$$

$$s^{ikl} := -\frac{1}{e} \frac{\delta L_M}{\delta \omega_{iAB}} e^k{}_A e^l{}_B. \qquad (79)$$

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[12]Here we follow the (slightly modified) notations used in [4]. They differ from those ones in [61] by sign in R_{ik} and T_{ikl}, and spin density defined in [4] is twice that one in [61] (Therefore, it appear in [109] and our formulas (74) and (75) the minus signs and in Eq. (75) the factor $1/2$).

Assuming the case of a Dirac matter field, where L_M is given by the expression,

$$L_D = \hbar c(\frac{i}{2}\overline{\psi}(\gamma^i\nabla_i - \overleftarrow{\nabla}_i\gamma^i)\psi - m\overline{\psi}\psi), \tag{80}$$

the source terms read (see, e.g., [61] and [4]),

$$T^A{}_i = \frac{\hbar c}{2}\overline{\psi}(i\gamma^A\nabla_i - \overleftarrow{\nabla}_i i\gamma^A)\psi, \tag{81}$$

$$s^{ikl} = s^{[ikl]} = \frac{\hbar c}{2}\overline{\psi}\gamma^{[i}\gamma^k\gamma^{l]}\psi = \frac{\hbar c}{2}\epsilon^{ikls}\overline{\psi}\gamma_s\gamma_5\psi, \tag{82}$$

where ϵ^{ikls} is the Levi-Civita symbol and $\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3$.

Due to the total anti-symmetry of the spin density and the field equations (77), the modified torsion T_{ikl} and the torsion S_{ikl} itself are also completely anti-symmetric. Therefore, because of (71), one has $\Gamma^k_{(ij)} = \left\{{k \atop ij}\right\}$. (It should be mentioned that, for the total antisymmetry of S_{ikl}, the Dirac equations following by varying the Lagrangian (80) with respect to ψ and $\overline{\psi}$ couple only to metric but not to torsion (see [4]).)

Because of the second field equation (77) one can substitute the spin density for the torsion and this way arrive at effective Einstein equations. For this purpose, we split the Ricci tensor into the Riemannian and the non-Riemannian parts (^{0} denotes Riemannian quantities and $_;$ the covariant derivative with respect to $\{\}$),

$$R_{Bi} = e^l_A R^A{}_{Bli} = \tag{83}$$

$$= e^l_A(\overset{0}{R}{}^A{}_{Bli} - S^A{}_{Bl;i} + S^A{}_{Bi;l} - S^C{}_{Bl}S^A{}_{Ci} + S^C{}_{Bi}S^A{}_{Cl}) =$$

$$= \overset{0}{R}_{Bi} + S^l{}_{Bi;l}.$$

Regarding that the Ricci scalar is given as

$$R = e^l_A e^{Bi} R^A{}_{Bli} = \overset{0}{R} + S_{CAD}S^{ACD} \tag{84}$$

we obtain Einstein tensor

$$R_{Bi} - \frac{1}{2}e_{Bi}R = \overset{0}{R}_{Bi} - \frac{1}{2}e_{Bi}\overset{0}{R} + S^l{}_{Bi;l} - \frac{1}{2}e_{Bi}S_{CAD}S^{ACD} \tag{85}$$

such that the effective Einstein equations have the form

$$\overset{0}{R}_{Bi} - \frac{1}{2}e_{Bi}\overset{0}{R} - \frac{\kappa^2}{2}e_{Bi}s_{klm}s^{klm} = \tag{86}$$

$$= -\kappa\left(\overset{0}{T}{}^D_{Bi} - \kappa\frac{i\hbar c}{8}\overline{\psi}\gamma^k\sigma^{lm}s_{klm}\psi e_{Bi} - \frac{1}{2}s^l{}_{Bi;l}\right)$$

$(\overset{0}{T}{}^D_{Bi}$ is the energy-momentum tensor of the Dirac field in the Riemannian space and $\sigma^{lm} = [\gamma^l, \gamma^m]$.) These equations differ by 3 terms from the Einstein equations:

i) On the left-hand side, it appears a cosmological term, where the effective cosmological constant Λ in the epoch under consideration, is given by the product: $1/2\times$ square of gravitational constant $\times$ square of the spin density of matter in this epoch. Due to the structure of s_{klm} given by Eq. (82), this term can be rewritten as follows[13]

$$-\frac{\kappa^2}{2}s_{klm}s^{klm} = -\frac{\kappa^2}{8}(\epsilon_{klms}\overline{\psi}\gamma^s\gamma_5\psi)(\epsilon^{klmt}\overline{\psi}\gamma_t\gamma_5\psi) = \tag{87}$$

$$= \frac{3\kappa^2}{8}\delta^t_s\psi^+\gamma_5\gamma^s\gamma_5\psi\psi^+\gamma_5\gamma_t\gamma_5\psi =$$

$$= \frac{3\kappa^2}{8}\psi^+\gamma^s\gamma_s\psi|\psi|^2 = \frac{3\kappa^2}{2}|\psi|^4$$

where $|\psi|^2 = \psi^+\psi$. In other words, for a finite fermion number, the effective cosmological term is proportional to the square of the fermion density. Thus, a cosmological term can be dynamically induced or, if such a term is assumed to exist at the very beginning in the equations, tuned away in an effective Einstein-Cartan-Kibble-Sciama theory. (For other approaches, there is a vast number of papers that one has to regard. For the approach of effective scalar-tensor theories, see, e. g., [106] and the literature cited therein.)

ii) On the right-hand side, it appears an additional source term given by the spin density s_{klm} and a divergence term. Originally, the latter is also a pure geometric term, namely the divergence of the torsion $-S^l{}_{Bi;l}$. It has the same property as the dark-matter terms in fourth-order metric theory (Sec. 3.1) and the tetrad theory (Sec. 3.2):

$$S^l{}_{Bi;l}{}^i = -g_{si}S_B{}^{ls}{}_{;l}{}^i = -S_B{}^{ls}{}_{;ls} = 0. \tag{88}$$

But, in contrast to the above-discussed cases, in virtue of the second field equation (77), it can be rewritten in a visible-matter term.

There is still another way to go over from the Einstein-Cartan-Kibble-Sciama theory to an effective GRT. Instead of combining the field equations (74) and (75), this way performs the substitution $S_{ikl} \rightarrow \omega_{ikl}$ in the Lagrangian

$$L(e^A_i, \partial e^a_i, S_{iAB}, \partial S_{iAB}, \psi, \partial\psi, \overline{\psi}, \partial\overline{\psi}) = \tag{89}$$
$$= eR(e^A_i, \partial e^A_i, S_{iAB}, \partial S_{iAB}) + L_D(e^A_i, \partial e^A_i, S_{iAB}, \psi, \partial\psi, \overline{\psi}, \partial\overline{\psi})$$

---

[13]$\psi^+ = \overline{\psi}^*$ (complex-conjugate and transposed ψ)

where L_D is given by Eq. (80). This provides the effective Lagrangian

$$L^*(e_i^A, \partial e_i^A, \psi, \partial\psi, \overline{\psi}, \partial\overline{\psi}) = eR^*(e_i^A, \partial e_i^A) + \tag{90}$$
$$+2\kappa L_D^*(e_i^A, \partial e_i^A, S_{iAB}, \psi, \partial\psi, \overline{\psi}, \partial\overline{\psi})$$

$$R^*(e_i^A, \partial e_i^A) = \overset{0}{R} - \omega_{ikl}\omega^{ikl} = \tag{91}$$
$$= \overset{0}{R} - S_{ikl}S^{ikl} = \overset{0}{R} - \frac{\kappa^2}{4}s_{ikl}s^{ikl} = \overset{0}{R} + \frac{3\kappa^2}{2}|\psi|^4$$

$$L_D^*(e_i^A, \partial e_i^A, S_{iAB}, \psi, \partial\psi, \overline{\psi}, \partial\overline{\psi}) = \overset{0}{L}_D + \frac{1}{4}i\hbar ce(\overline{\psi}\gamma^i\sigma^{AB}\omega_{iAB}\psi) = \tag{92}$$

$$= \overset{0}{L}_D + \frac{1}{4}i\hbar ce(\overline{\psi}\gamma^i\sigma^{AB}S_{iAB}\psi) =$$

$$= \overset{0}{L}_D - \frac{1}{8}i\hbar ce(\overline{\psi}\gamma^i\sigma^{AB}s_{iAB}\psi).$$

The variation of L^* with respect to e_i^A provides the equations,

$$\overset{0}{R}_{Bi} - \frac{1}{2}e_{Bi}\overset{0}{R} - \frac{3\kappa^2}{2}e_{Bi}|\psi|^4 = -2\kappa\frac{1}{e}\frac{\delta L_D^*}{\delta e_k^B}g_{ki} \tag{93}$$

where

$$\frac{1}{e}\frac{\delta L_D^*}{\delta e_i^A} = (\overset{0}{T}{}^D)_A^i - \frac{1}{8}\frac{\kappa i\hbar c}{e}\frac{\delta e}{\delta e_i^A}(\overline{\psi}\gamma^c\sigma^{AB}s_{cAB}\psi) = \tag{94}$$

$$= (\overset{0}{T}{}^D)_A^i - \frac{1}{8}i\kappa\hbar c(\overline{\psi}\gamma^c\sigma^{AB}s_{cAB}\psi)e_A^i$$

such that , finally, one has:

$$\overset{0}{R}_{Bi} - \frac{1}{2}e_{Bi}\overset{0}{R} - \kappa^2 e_{Bi}\left(\frac{3}{2}|\psi|^4 - \frac{1}{4}i\hbar c(\overline{\psi}\gamma^c\sigma^{AD}s_{cAD}\psi)\right) = -\kappa\overset{0}{T}{}^D_{Bi}. \tag{95}$$

In contrast to Eq. (86), in Eq. (95) the term proportional to s_{klm} is absorbed in the effective cosmological term, while the "wattless" term is missing. But, generally, one meets again the above-described situation. To some extent, the effective theory represents a general-relativistic generalization of v. Seeliger's ansatz (4).

4. Conclusion

As introductorily mentioned, the idea of going beyond Riemannian geometry can be motivated differently. In dependence on the chosen approach, the question as to the matter coupled to gravitation has be answered in a

different manner. As long as one wants to reach a unified geometric theory, there is no room left at all for any matter sources. Since all matter was to be described geometrically such sources are out of place. However, if one interprets the additional geometric quantities as describing gravitation, then one has to introduce matter sources. If one continues to assume the validity of SEP in this case, one has to confine oneself to the consideration of those purely metric theories which were reviewed in Sec. 3.1. In particular, this means that the source of gravitation is assumed to be the metric energy-momentum tensor. If one requires that the theory is derivable from a variation principle this follows automatically. This mathematical automatism has a deep physical meaning.

To make the latter point evident let us return to the discussion of SEP. The formulation of SEP given above focuses its attention on the coupling of gravitation to matter sources, without saying anything on the structure of the matter sources. However, reminding the starting point of the principle of equivalence, namely the Newtonian equivalence between inertial, passive gravitational and active gravitational masses, the SEP implies also a condition on the matter source. Indeed, while the EEP is the relativistic generalization of the equality of inertial and passive gravitational masses, the SEP generalizes the equality of all three masses. Therefore, the special-relativistic equivalent of the Newtonian inertial mass, i.e., the symmetrized special-relativistic energy-momentum tensor, has to be lifted into the curved space and chosen as relativistic equivalent of the active gravitational mass, that means, as source term in the gravitational equations. Of course, this condition is automatically satisfied if one starts from a Lagrangian in a Riemannian space. Therefore, assuming that EEP necessarily leads to a Riemannian space, in [138] this aspect of EEP was not mentioned explicitly. But if one considers theories based on non-Riemannian geometry one should keep in mind this condition required by SEP for the source term.

As was shown in Sec. 3.2, the latter limitation on matter can also be imposed on tetrad theories of gravity. However, it loses its meaning if one goes over to more general geometries and theories of gravity, respectively. In the case that the Lagrangian is given by the Ricci scalar this even leads to a trivialization of the geometric generalization [127]. Therefore, it is more consequent to consider non-Riemannian geometry and its perspectives for a generalization of GRT from the standpoint of the matter sources, as it was done in [111, 112, 68, 131, 61, 62].

As the examples given above demonstrate, in post-Einsteinian theories of gravity there are typical non-Einsteinian effects of gravitation which correspond to certain pre-relativistic ansatzes. One meets absorption (or/and self-absorption) and suppression (and/or amplification) effects.

In case that the additions (Θ_{ik}, H_{ik}, and S_{Bki}, respectively) to the matter source, have a suitable sign there exists an amplification of the matter source by its own gravitational field. Above we called these terms "hidden- or dark-matter terms". To some extent, this is justified by the following argument [13].

The fact that on large scales there is a discrepancy in the mass-to-light ratios can be explained in two alternative ways. Either one presupposes the validity of the theory of GRT and thus, in the classical approximation, i.e., for weak fields (where $GM/rc^2 \ll 1$) and low velocities (where $v \ll c$), the validity of Newtonian gravitational theory on large scales. Then one has to assume that there exist halos of dark matter which are responsible for this discrepancy. Or one takes a modification of GRT into consideration. Then, the classical approximation of the modified theory should represent a gravitational mechanics that is different from the Newtonian one. For, since the dynamical determination of masses of astrophysical systems is always performed in the classical approximation, this approximate mechanics has to work with a gravitational potential deviating from the usual $-GM/r$ form at large distances. From this view larger mass parameters seem to be responsible for the observed motions in astrophysical systems[14].

Interestingly, the above-discussed pure gravitation-field additions have one essential property of dark matter. They can be interpreted as source terms whose divergence vanishes. Accordingly, they do not couple to the optically visible matter and are themselves optically invisible. Of course, to answer the question whether they can really explain the astrophysical data one has to study the corresponding solutions of the respective gravitational equations.

To summarize, when one presupposes the framework of GRT in order to interpret the results of measurements or observations, then one finds the following situation: In all three examples there are dark-matter effects. In the case of tetrad theory, as an implication of the potential-like coupling, additionally one finds a variable active gravitational mass and a variable gravitational number G, respectively. In the case of the Einstein-Cartan-Kibble-Sciama model one finds a Λ-term simulated by spinorial matter. Thus, if one measures a variable G number one has an argument in favor of a theory with potential-like coupling. Moreover, dark-matter or Λ-effects do not necessarily mean that one has to search for exotic particles or that one has got an information about the value of Λ in GRT. These effects could also signal the need for a transition to an alternative theory of gravity.

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[14]Another approach to a modified Newtonian law is given by the MOND model. (For this see [108] and the contribution of V. DeSabbata in this volume.)

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